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The nuclear shell model, the cosmic abundances and why we do not know how the heavy elements were synthesized

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Abstract. We review the history of the nuclear shell model and explain the present day situation why we still do not know how the heavy elements were synthesized.

1. Introduction

We first review the evolution of the fundamental concept of the nuclear shell model and it relation to the cosmic abundance. Next we discuss the general road map of nuclear astrophysics and the fundamentals of neutron capture processes. Finally we search the reasons why despite ~ 7500 papers on the subject we still do not know how the heavier than iron elements were synthesized. The present talk is a synopsis from the book *The Quest for Chemical Element Genesis and What it Tells Us about the Universe*, by Giora Shaviv, Springer 2011, in press.

2. Cosmic abundances as signals to nuclear structure

Victor Goldschmidt (1888-1947) was the first to notice the peculiar curve of abundances and its unique peaks (Goldschmidt, 1930). He found that certain elements have significantly higher abundances than neighboring elements. These elements did not show any chemical similarities and hence, he inferred the fundamental conclusion that the difference in abundance is due to nuclear structure and not chemical reactions. He classified the elements according to Z and N and not according to A=Z+N. The abundance peaks he discovered were at: Z=28, 40, 50, 74, 82, and 90, and at N= 30, 50,82, and 108. Inaccuracies in the data prevented Goldschmidt from concluding that the numbers are identical for protons and neutrons. Goldschmidt did not live to learn about the explanation of his peculiar nuclear numbers but his colleague Suess (Suess, 1947)[1] dis-

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[1] Goldschmidt as a jew, resigned in 1935 from his Göttingen professorship in protest against the treatment of 'non-Aryans' and returned to Norway. Suess was involved in the german uranium project and suggested the use of heavy water as a moderator to slow down the neutrons in uranium fission reactors and thus increase the absorption of neutrons by the uranium. This suggestion brought him to Norway, where the germans used the hydro-electric power plant to produce heavy water of which he was a consultant. Being in Norway he met Goldschmidt and it is during these discussions that Suess realized the meaning of the special numbers discovered by Goldschmidt. Few days after Suess visit, Goldschmidt was caught by the Gestapo and put of a freighter to be deported. Just before leaving harbor a Norwegian police officer came to claim his release with the excuse of being important for Norwegian industry. The next day he was smuggled to Sweden

cussed the abundance curve with its ′peculiar numbers′. The numbers Suess got were: For neutrons: 20, 28, 50, (58), 82 and for protons: (20), 26 or 28, 50, 74, and 82. Note that the magic numbers for neutrons and protons as they appear in this list, are not identical. The same ′special numbers′ were discovered by the nuclear experimentalist Haxel.

In the nucleus, it is the mutual attractive force acting between all particles, which governs the motion and hence, there is no a priori reason to expect any parallelism between the structure of the atom and that of the nucleus. While in the solar system and in the atom, one can assume that the interactions between the planets and themselves, or between the electrons and themselves, are small and hence, can be ignored as a first step this is not expected to be the case in the nucleus.

Yet, and against all physical logic, the history of nuclear theory is a story of wandering between two extremes: the shell model and the collective model. All the physical arguments are in favor of the collective model, a model in which each nucleon feels all the others in contrast with the shell model, where one assumes that there exists ′an effective central force′ and the nucleons move under the effect of this ′fictitious′ force. The collective model predicts that the properties of the nuclei should be a smooth function of the number of particles in the nucleus. However, the abundances of the elements, as well as many other properties of the nucleus, are neither monotonic as a function of the atomic number Z; nor of atomic weight A; nor of the number of neutrons N; and provide strong evidence in favor of the simple, though completely against the straight-minded physical intuition, picture of individual particles moving under a central force.

Nuclear theory did not really exist before the discovery of the neutron in 1932 and consequently whatever theory there was had to fight with two problems: (a) the stability problem of the nucleus: How the positive charges hold together and do not dissolve the nucleus into its constituents, and (b) the difference between the number of positive charges (protons) and the total mass of the nucleus. The simple solution was to assume that the nucleus is full with protons and invent ′nuclear electrons′ which neutralize the extra protons leaving only Z unneutralized protons. This of course does not explain why the nucleus does not split into its constituents. The idea of a nuclear force between the particles in the nucleus was not born yet. We jump over the initial attempts to create a theory of the nucleus and get directly to the first collective model by Gamow.

3. Gamow 1929

In 1928 Gamow was still a graduate student in Leningrad and upon a special recommendation of senior russian physicists was allowed by Moscow to spend the summer in the Max Born Institute in Göttingen. During his stay he developed the α-decay theory as well as theory of tunneling (Gamow, 1970). In 1929 Gamow participated in a discussion in the Royal Society (Gamow, 1929a,b) about the physics of the nucleus and proposed a ‘simple model’ for the nucleus, which he called ‘the water drop’ model. As for the basic building block, Gamow argued that a certain number of protons (not more than three) and electrons can be bound to an α-aggregate without forming a new α-particle. Gamow claimed that such a unit is less tightly bound than a nucleus with only α particles, in other words, the fundamental particles of the liquid are α-particles. The first model Gamow treated was made only of α particles and assumed that there is an attraction force between the α particles. Gamow assumed that the nuclear force has a very short distance so that the interaction distance is some r^*. *The sphere of radius* r^* *is well known in the theory of capillarity as the sphere of molecular action. We can say that the particle inside the liquid has no resultant force acting on it if the distance from the boundary is greater than* r^*. A nucleon inside feels a force from all directions while a particle near the surface feels a force only from inside. The result is an effective force which acts like surface tension. *Such a collection of* α *particles will be very like a minute drop of water where the inside pressure,*

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and died in London. Suess’ paper was essentially an obituary to Goldschmidt.

due to the kinetic energy of quantized motion, is in equilibrium with the forces of surface tension trying to diminish the drop radius. The important point for the nuclear drop model is the question of the quantum number to be ascribed to the different α particles in the drop. Gamow's solution: *all α particles in the nucleus must be considered to be in the same state with quantum number unity* as the α particles are bosons.

In the wake of Gamow's theory of α-decay he was convinced that the nucleus is composed of α particles (Gamow (1930), in other words, Harkins' basic theory. Actually, this was one of the main topics in the book on nuclear physics Gamow published in 1931 (Gamow, 1931), before the neutron was discovered. When in 1928 Gamow went to Bohr in Copenhagen do discussed the nucleus (Gamow, 1928) he assumed a collection of interacting α particles.

The metaphor of water drops includes also the assumption of non-compressibility of the nucleus which in turn implied that the radius of the nucleus varies as $A^{1/3}$, which was by then a known result from α scattering. Together with the assumption of surface tension, Gamow was able to predict the existence of what later became known as the valley of stability. Effectively, he was the first to obtained the first two terms in the known phenomenological formula for the binding energy of nuclei (Gamow, 1930).

4. Signs of non-smoothness in nuclear properties

In 1930, just before the discovery of the neutron, Barton (1929, 1930) discovered *A new regularity in the list of existing nuclei* when he plotted the number of protons in the nucleus, P, against the number of electrons in the nucleus, E. The atomic number is $Z = P - E$. Barton noticed that the nuclear data plotted in this way does not produce a smooth curve but three distinct ′clusters′ of nuclei. Moreover, the clusters appeared symmetrical with respect to their center. If so, suggested Barton, an analogous symmetry should be exhibited by the relative abundances of the nuclei, thus indicating probably for the first time, a connection between nuclear properties and abundance.

Bartlett (1932a,b,c) attempted to explain Barton's clusters by assuming that there exist closed shells in the nucleus. *The center of the cluster seems to lie about where the shells would be half-completed provided that the closed shells correspond to the masses 36,64,100,144, etc.* It was Bartlett who introduced the idea of nuclear shells.

In 1933 Landé (1933) demonstrated that in odd-Z nuclei the unpaired proton contributes the magnetic moment of the nucleus. Consequently, Schmidt and Schüler (1935) discovered nuclei with large deviations from spherical symmetry. The liquid drop model does not predict such large deviations. The particular success came when they discovered that the nucleus of europium is not spherically symmetric. Landé′*s* explanation used by Schmidt, was remarkably simple. Lande assumed that *one particle only, one proton or one neutron, is responsible for the total spin and magnetic properties of the whole nucleus.* In this way the total angular momentum is explained as the property of the last nucleon (Schüler & Schmidt, 1936), which surrounds a spherical core. Schüler and Schmidt did not discuss shells let alone closed ones. They discussed however, single particle moving outside a closed spherical core, and the issue was that a property of a single nucleon was associated with the property of the entire nucleus.

5. The birth of nuclear physics

In 1932 Chadwick discovered the neutron (Chadwick, 1932a,b) which gave rise to the solution of the problem of nuclear stability.

6. Elsasser and Guggenheimer

Elsasser (1904-1991) and Guggenheimer were both in Paris, refugees from Nazi Germany but interested in nuclear physics from different points of view. Progress came when Elsasser (1933, 1934, 1935) and Guggenheimer (1934), motivated by the idea and the controversy that the nucleus contains α particles, looked into the problem. Guggenheimer, as a chemist, searched for nuclear analogy with the atomic periodic table. Elsasser, on the other hand, noticed the existence of ′special numbers′ of neutrons and protons which endowed the corresponding nuclei a particularly stable configuration. In analogy with atomic electrons, he correlated these numbers with closed shells in a model of non-interacting nucleons obeying Pauli's exclusion principle and occupying energy levels generated by a potential well. In parallel, Guggenheimer tried to classify the nuclei according to the number of protons and neutrons and to show that the α model does not work. The periodicities so found were interpreted as reflecting full nuclear shells. Evidence for the magic numbers at N=50 and N=82 became evident. The N= 28 was less clear. These hypotheses were not pursued any further at that time, both because of the apparent paradox that strong inter-nucleon forces would average out in such a simple way, and the paucity of experimental data in favor of a single-particle description. The lack of rigorous derivation and what looked to many as numerology, caused the community not to take these findings sufficiently seriously. Robert Oppenheimer, as reported by Willy Fowler (1983), was very skeptical of Elsasser's work because Elsasser could not fit the data beyond 20 nucleons. The next special number that Elsasser got was 40. But in the experiment, the next special properties occur at 50. Oppenheimer expressed his doubt by telling Jensen that: *Maria and you are trying to explain magic by miracles*[2].

The discoveries by Elsasser and Guggenheimer were rejected by Bethe and Bacher in Bethe's ′Bible′ of nuclear physics (Bethe & Bacher, 1936), who claimed that although the order of single nucleon orbits proposed by Elsasser and others reproduced the ′special numbers′ , their model lacks a *theoretical foundation*. A deeper argument which they presented, concerns the effect of nucleon-nucleon interactions on the single nucleon picture of the shell model. It is fair to admit that this problem has remained with us until now. It is in this aspect that the shell model still *lacks theoretical foundation*.

In a way, Elsasser used all the data that was known at that time. Hence, until the late nineteen forties, when much more data on various nuclear properties, in particular β-decays and nuclear moments like the magnetic one, accumulated, nothing happened that could advance the solution of this problem.

The 1938 Nobel Prize for Physics was awarded to Fermi for his work on *Artificial Radioactivity Produced by Neutron Bombardment* (Fermi et al., 1934; Amaldi & Fermi, 1936), and for nuclear reactions brought about by slow neutrons (Fermi, 1934). The Nobel prize was not awarded for the ingenious β-decay theory, though his theory of β-decay is mentioned in the Nobel publication, or the Fermi-Dirac statistics, the importance of both definitely commensurates with the neutron capture work if not more.

As for our issue here, in his Nobel address Fermi stressed that the capture probability of neutrons by various nuclei *varies with no apparent regularity for different elements* by a factor of thousands. Fermi offered no explanation. On the other hand, the sharp and closely spaced resonances discovered in the neutron capture, were the trigger for Bohr's theory of the compound nucleus (see next section). The large variations in the neutron capture rate just hinted towards the independent particle model and the existence of closed neutron shells but nobody at that time actually deciphered the hint. The particular importance of these experiments is that they directed physicists in later years towards two extreme modes, the compound and the shell modes, cf. fig. 1.

Interesting to note, D'Agostino, the chemist in Fermi's group that participated in the neutron capture experiments was convinced that nuclei had a periodic system like atoms. The group disagreed and ridiculed him (Segrè, 1981). Thus, there was a non-believer in Fermi's camp who turned out to be right.

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[2] Jensen, Nobel address, 1963. The term 'Magic numbers' was coined by Wigner to express his contempt (at the time) to the idea.

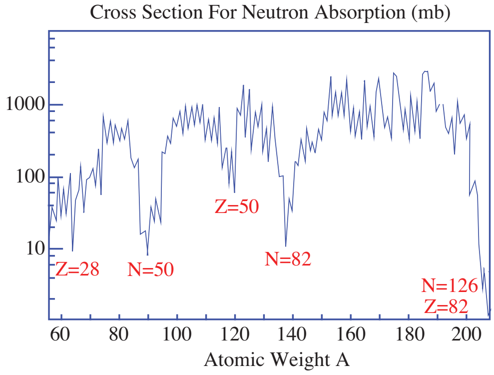

Fig. 1. Neutron capture cross section as a function of atomic weight A.

7. Bohr's declaration against the independent particle model

On January 27, 1936 Bohr delivered before the Copenhagen Academy a lecture about *Neutron capture and Nuclear Constitution*. The lecture, which was later published in Nature (Bohr, 1936), was a crusade against the independent particles model and propaganda for the collective model. No doubt, from a strict physical point of view Bohr's brilliant idea was right, but nature plays sometimes havoc with physicists. Bohr observed that the neutron capture resonances, as discovered by Fermi, were narrow in energy. Using the Heisenberg uncertainty principle, Bohr calculated that the time involved is several orders of magnitude longer than 10^{-21} *sec*, which is the time needed for the slow neutron to cross the nucleus. Hence, the nucleus, into which a large amount of energy was deposited by the penetrating neutron, must stay a relatively very long time in an excited state before it finally decays and yields the products of the reaction. Bohr and Kalckar (1937a)[3] and Bohr (1937) claimed that *every nuclear transmutation will involve an intermediate stage in which the energy is temporarily stored in some closely coupled motion of all particles of the compound nucleus.* In other words, Bohr envisioned the nuclear reaction as

$$initial\ state \rightarrow (compound\ state) \rightarrow final\ state.$$

Bohr claimed that: *It is, at any rate, clear that the nuclear models hitherto treated in detail are unsuited to account for the typical properties of nuclei for which, as we have seen, energy exchanges between the individual nuclear particles is a decisive factor. In fact, in these models it is, for the sake of simplicity, assumed that the state of motion of each particle in the nucleus can, in the first approximation, be treated as taking place in a conservative field of force, and can therefore be characterized by quantum numbers in a similar way to the motion of an electron in an ordinary atom.* The fundamental assumption of the shell model is that each nucleon moves in a potential well produced by all nucleons and except for this mean smooth force, the nucleon is only marginally affected by the rest of the nucleons. This supposition apparently contradicts the basic premise in nuclear physics, claimed Bohr. The interaction between the nucleons is very strong and hence, fast energy exchange between them takes place. Bohr continued with a carefully formulated, and what intended to be a calamitous blow to the shell model, statement: *In the atom and in the nucleus we have indeed to do with two extreme cases of mechanical many-body problems for which a procedure of approximation resting on a combination of one-body problems, so effective in the former case, loses any validity in the latter*. In short, do not mix the opposing descriptions.

8. Support in question - The Breit-Wigner formula

Completely independently and even slightly before the appearance of Bohr's Nature paper, Breit (1899-1981) and Wigner (1902-1995) (Breit & Wigner, 1936) attacked the same problem and provided a beautiful mathematical formulation[4]. As a matter of fact, Bethe (1935), Fermi and his group (Amaldi et al., 1935), Perrin and Elsasser (1935) Beck and

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[3] *On the transmutation of atomic nuclei by impact of material particles. I.*

[4] Bohr's paper (1936) in Nature was mainly argumentative typical to the deep thinker Bohr, as the attempts at a mathematical formulation by him and Kalckar, were delayed. The paper by Bohr and Kalckar (1937a) came out significantly later.

Horsley (1935) proposed alternative explanations. Breit and Wigner rejected all these explanations claiming that *the combined evidence of experimental results and theoretical expectation is thus against a literal acceptance of the current theories* and attacked the problem of the resonances observed in slow neutron capture. They came out with the famous Breit-Wigner formula, which was later generally reinterpreted as the mathematical expression for Bohr's compound nucleus idea. But one has to read carefully Breit and Wigner, who wrote in the abstract to their paper that: *These facts can be accounted for by supposing that in addition to the usual effect there exist transitions to virtual excitation states of the nucleus in which not only the captured neutron but, in addition to this, one of the particles of the original nucleus is in an excited state.* In other words, Breit and Wigner did not suppose that all the nucleons share the energy of the incoming neutron but only one. Moreover, the single excited nucleon in the nucleus was assumed to stay inside a potential well in a well defined quantum state. Apparently, the assumed two phases in Breit and Wigner formalism, namely the entrance and the exit channel, were interpreted as support for Bohr's ′intermediate state′ idea.

9. The compound model domination

Soon after the lecture in Copenhagen, which was more of an address than a lecture, Bohr went on a 6 months international sales-tour ′marketing′ his model, even before he had a concrete mathematical scheme for it (Bohr & Kalckar, 1937b). While Bohr and Kalckar criticized Schmidt and Schlüter's independent particle model, they did not offer an explanation as to why the properties of the nuclei are not smooth as a function of the atomic weight. A major factor in the wide spread belief in the dogma was the massive support Bohr's compound model obtained from Bethe and Bacher in Bethe's ′Bible′. This review had a pervading influence on physicists and won its name justifiably. Bethe extended Bohr's theory assuming that the nuclei are perfect-many-body-systems. It should be stated that Bethe and Bacher provided a very extensive review of Elsasser's model. Bethe even contributed some additional evidence in favor of the model. But the authors concluded with a final negative verdict: *In conclusion, we want to emphasize again that reliable conclusions about the shell structure of nuclei can only be drawn when atomic weight determinations will be available which are guaranteed to at least three decimals, i.e. 1 part in 100,000 for atomic weights of the order of 100* (Bethe, 1937)[5]. In other words, the masses of the nuclei claimed the authors, are not sufficiently accurate to determine whether the masses are a smooth function of the atomic weight or not. Moreover, strong arguments are presented in the ′Bible′ that spin-orbit coupling should be very weak (wait and see later how the situation changed).

The compound model's greatest success is in the Bohr and Wheeler theory of nuclear fission (Bohr & Wheeler, 1939)[6]. But the compound model has its shortcoming. According to this model, the valley of stability should be a smooth valley without small cliffs. But we see steep slopes at, for example, N=128, namely, very large changes in the binding energy upon the addition of one neutron or one proton. This cliff was called by Gamow ′Heisen-Berg′[7] but it did not change his full support of Bohr's theory. On the other hand, the problem how to accommodate the entire list of magic numbers with a single potential or theory remained unsolved. The wide spread belief in Bohr's theory treated the magic numbers as an irrelevant

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[5] This is the 2nd volume of the Bible and it is devoted to theoretical nuclear physics and was written by Bethe only.

[6] Bohr and Wheeler did not mention in their 1939 paper Gamow's priority of the idea of a liquid drop from 1928-1930, though Bohr was for sure aware of it as Gamow was in Copenhagen at the time the idea was born. The injustice was corrected only for history, when Wheeler in his *Oral History Transcript Interview by K.W. Ford, Princeton University January 12, 1994* said that: *That thought* (of Gamow) *had fallen in abeyance in the meantime*. Better late than never. Bethe and Bacher, 1936, did not mention Gamow's priority on the liquid drop models, and Gamow and Bethe were good friends ...

[7] The Heisenberg mountain

curiosity or numerology which has no room in physics.

The impact of Bohr's lecture and Nature paper on nuclear physics theory was to suppress practically all research assuming the independent particle model and any variation of it and to deter young physicists from pursuing 'non-physical' research. Only those who were deeply involved and already eminent, like Wigner and Hund, were not discouraged and continued their exploration into nuclear structure using the independent particle model, whenever the compound model had no appropriate answer. Their work, however, was mostly limited to nuclei lighter than $N, Z \leq 20$, for which the order of single nucleon orbits could be simply understood[8]. Take for example young Victor Weisskopf (1991)[9] who stated that: *Under Bohr's influence I was thinking about the compound nucleus.* Another young physicist who subscribed to the compound model was Robert Oppenheimer (Kalckar, Oppenheimer & Serber, 1937), who applied right away, with Bohr's collaborator Kalckar (1910-1938), the idea of the compound nucleus to another nuclear phenomenon[10]. The pressure against the shell model supporters was not trivial. Gamow met Elsasser in Paris and explained to him that he risks his prospects in getting a job in physics had he chosen to continue exploring the individual model (Johnson, 1992). Indeed, Elsasser moved to geophysics and is known for the theory of the dynamo theory of the Earth's magnetic field.

Also Rose and Bethe (1937) tried to follow Bethe and Bacher's suggestion written in 'Bethe's Bible', namely: *the individual particle model affords one the opportunity to construct a rational theory of nuclear spin and magnetic moments for light nuclei.* This is exactly what Rose and Bethe tried to do in the paper. The authors, who applied the basic results of Feenberg (1906-1977) and Wigner (Feenberg & Wigner, 1937) used essentially spectroscopic methods developed to handle problems with many electrons in the atom[11]. They assumed S-J and L-J coupling and discussed the motion of particles in well defined quantum levels. The compound model did not allow for single particles with well defined quantum numbers. However, they added that this is an approximation. Elsasser was not mentioned.

In 1938 Gamow published his book *Structure of Atomic Nuclei and Nuclear Transformation* (Gamow, 1938). In a nut shell, here is what Gamow thought about the individual particle model: *One may hope that further investigation along these lines will add considerably to our understanding of more detailed problems of structure. Much has already been done with rather overlapping results by Barlett, Gapon, Ivanenko, Elsasser, Guggenheimer and others; it is not referred to in detail here because the author was never able in studying these articles to remember the beginning when he was reading the end* [12] Peierls (1940) wrote a long review on Bohr's theory of nuclear reactions without mentioning at all the problems of magic numbers. You had to be a genuine iconoclast to work on the independent particle model in those days and ready to risk your career. Research in the shell model was largely suppressed until the late nineteen forties when more problematic data to the compound model was uncovered, on one hand, and Mayer, Jansen, Suess (1909-1993) and Haxel

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[8] Wigner admitted that he really did not believe in the independent particle model of Elsasser but changed his mind when Mayer and Teller told him about it after WWII. Wigner then claimed that Hund's results were similar to those of Elsasser and nobody understood them. Interview of Wigner by C. Weiner and J. Mehra on 1966, Niels Bohr Library & Archives, American Institute of Physics, College Park, MD USA, www.aip.org/history/ohilist/LINK

[9] *Fifty-five years of my life in nuclear physics*

[10] The authors referred to a *N. Bohr, Science, to be published* paper. It appeared later in 1937

[11] They assumed the Russell-Saunders coupling.

[12] The irony is that the type of states defined by Gamow in 1928, namely states which are unstable and decay (the energy has an imaginary part) are called Gamow states and so you can find papers discussing the 'Gamow states in the shell model'. See for example Michel, Nazarewicz, Płoszajczak & Okołowicz (2003) "*Gamow shell model description of weakly bound nuclei and unbound nuclear states*".

violated the boycott on the independent particle idea, on the other hand.

10. From cosmic abundances to nuclear structure

The evidence for a unique cosmic abundance of elements curve was already conclusive towards the late nineteen forties. Consequently, Mayer and Teller (1949) [13] attempted to explain the abundances by means of the polyneutron theory. The theory will not be discussed here but its failure was the trigger for the shell model. In her 1963 Nobel lecture, Mayer recounted that it was during the work on the abundances of the elements that she stumbled upon the magic numbers. *We found that there were few nuclei which had a greater isotopic as well as cosmic abundance than our theory [...] could possibly explain*[14]. It soon turned out that the polyneutron theory was wrong on many counts and it was its failure which led Mayer to propose the nuclear shell model.

The first paper by Mayer (1948) [15] summarized experimental data showing that nuclei with 20, 50, 82 and 126 neutrons or protons are particularly stable. Interesting for element synthesis, Mayer pointed to the particularly low neutron capture probability by nuclei possessing a magic number of neutron. It is these probabilities which control the synthesis of the elements by the rapid neutron capture process. Mayer mentioned the fact that shell closure at $N = 20$ is understood but she did not speculate about the other numbers. Mayer also explained why the identification of the atomic electronic closed shells is so easy while the identification of the nuclear closed shell is so difficult. The ionization potential at a closed shell varies by several hundred percent, while the binding energy of the last nucleon at a nuclear closed shell varies by not more than 30%. Are 30% worth all the ado? Apparently yes!

Mayer's paper triggered two very interesting and important papers by Feenberg and Hammack (1949)[16] and by Nordheim (1949)[17] which were published several months after Mayer's paper was published. The papers arrived to the editor on the same day and were printed back to back. Also, copies of both papers were sent to Mayer before publication. The papers were crucial in encouraging Mayer that she was on the right track. What was important for Mayer was that Feenberg and Hammack, as well as Nordheim, assumed that the nucleon outside the closed shell could be handled as a single particle with well defined quantum numbers. The question then was what is the force which orders the energy levels in such a way that the magic numbers correspond to full shells? More accurately, the energy levels are not evenly spaced. A group of different energy levels, the energy of which differs from one another by a relatively small amount, is called a shell. The difference in energy between the shells is relatively large compared to the differences in energy between the levels of the same shell.

To get an agreement with the magic numbers, Feenberg and Hammack (1949) assumed a simple rectangular potential well and got an agreement for $N, Z \leq 20$[18]. To obtain a good agreement for more massive nuclei they had to adopt Elsasser's ′a central depression to the well′ to form what is called the 'Wine bottle potential'. The data about the total angular momentum of nuclei tended to corroborate the shell model. Consider a core of nucleons with one nucleon outside the core. According to the shell picture, the core is spherical and does not rotate. So it has vanishing angular momentum. The total angular momentum of the nucleus would then be the total angular momentum of the single nucleon moving outside the core.

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[13] Mayer & Teller submitted June 1949. Note that although the results of this research were the trigger to the Mayer's shell model papers, they were submitted to the journal and published only after Mayer's first papers on the shell model.

[14] Goeppert Maria, M., *The shell model*, in Nobel Lectures, Physics, 1963-1970, Amsterdam, 1972

[15] Mayer submitted April 16, 1948 and published August 1, 1948

[16] Feenberg & Hammack submitted December 27, 1948.

[17] Nordheim submitted December 27, 1948.

[18] Actually, a wide range of potential wells yield a good agreement for less than 20 nucleons.

Feenberg and Hammack realized that a the ′bottle of wine shape′ potential has its energy levels arranged in the right way. What would cause a Wine Bottle Potential? The authors looked to the Coulomb repulsion between the protons as the source of a depression. Because of the Coulomb repulsion, they hypothesized, the density of the protons is minimal near the center and maximal near the boundary of the nucleus. The distorted proton distribution induces a distortion in the neutron distribution. The authors mentioned that this effect was investigated over a decade ago by Wigner (1940)[19] and Feenberg (1941) and more recently by Nordheim. Nordheim, on the other hand, made explicit use of the results of Schmidt (1937) for nuclear magnetic moments. The model of Landé (1934), in which *a single particle, one proton or one neutron, is responsible for the total spin and magnetic properties of the whole nucleus*, was taken up by Schmidt, who had by then more experimental data than were known to Elsasser. Both level schemes of Nordheim and Feenberg and Hammack, list the various orbits by their orbital angular momentum and characterize the states by the total L, and the total intrinsic spin, S like in the Russell-Saunders coupling of atomic electrons (LS-coupling).

A few months after the appearance of the two papers by Feenberg and Hammack and by Nordheim, appeared Mayer's breakthrough (Goeppert Mayer, 1949) when she assumed the S-L orbit coupling and what it does to the energy levels. Mayer had the privilege to discuss the paper with Fermi[20] who asked her whether there is an indication of strong S-L coupling. She replied instantly *Yes*[21]. Mayer thanked Fermi for *this remark which was at the origin of this paper*. However, the super imaginative Fermi was skeptical at the beginning but later changed his mind and believed in Mayer's model. Mayer's paper to the PRL was held by the editor who asked Feenberg, Hammack & Nordheim (1949) to compare the three proposed schemes. The two letters, that of Mayer and that of the referees, were published back to back.

All three models explain successfully the arrangement of up to 20 nucleons. The problem was with the larger number of nucleons, so they claimed that: *These facts suggest, however, that a rearrangement of levels may be successful.* Consequently three different rearrangements were suggested. Feenberg and Hammack suggested a change in the Coulomb force. Nordheim suggested a discrimination against levels with high angular momentum. Mayer's scheme followed the order of levels in a potential well and achieved the breaks at the right locations by assuming an arbitrary very strong spin-orbit coupling. *All three schemes give, of course, the empirical shell numbers and a statistical correlation with observed spins and moments*. In particular they wrote that: *The shell structure in nuclei, is however, so pronounced an effect that one may hope to obtain an interpretation even on the basis of such crude approximation as the individual model.* The comparison showed that the only differences were in the third shell and above. The differences are however, not in the number of particles in each shell but in the angular momentum of the levels composing the shells. The principle is the same in all three models, namely, add an agent which destroys the spherical symmetry and induces a situation where, by the lifted degeneracy of the simple potential, rearranges the energy levels in a new order under the condition of the existence of the magic number of nucleon in closed shells. It is now relatively simple to decide which model holds, simply check the angular momentum.

11. Enlightenment appears twice

Two months after Mayer's second paper appeared, the Physical Review published the paper by Haxel, Jensen & Suess (1949).[22] with

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[19] Wigner, E., Bicentennial Symposium, University of Pennsylvania, 1940.

[20] Mayer, M., in Nobel Lectures, Physics 1963-1970, Elsevier Pub. Co., Amsterdam, 1972.

[21] There are places where it is claimed that her reply was: *Yes, and it explains everything.*

[22] After WWII Suess succeeded to get the nuclear theoretician Jensen interested in the problem.

the title *On the 'Magic Numbers' in Nuclear structure*. The paper was preceded by a short paper (Jensen, Suess & Haxel, 1949a,b) in German. This was an independent discovery of the strong S-L effect in nuclear structure.

The history how Jensen and his collaborators reached their shell model is interesting. Few years earlier, in 1946, Hans, Jensen & Steinwedel (1946) noticed that the masses of the nuclei are not a smooth function of the proton or neutron number and they claimed that the data provides support for the α-particle model. In 1947, after the death of Goldschmidt, Jensen and Suess wrote a paper in the memory of Golschmidt (Jensen & Suess, 1947)[23]. The paper contained a comparison of the abundances with the differences between neutron number and proton number divided by their sum. Next, they tried to explain the observed curve as a consequence of thermodynamic equilibrium between the different species.

Suess and Haxel reached independently the conclusion that nuclei must exhibit closed shell phenomena: Suess examined the abundances and Haxel analyzed nuclear data. Jensen did not know what to do with the magic numbers and being under the influence of Bohr's 1937 paper, on one hand, and pressed by his two colleagues on the other hand, hesitated. It was arranged by fate that during a visit to Copenhagen, Jensen stumbled upon Mayer's first paper in the PRL and dared to discuss it in a seminar where he also presented his results. Bohr was present and heard Jensen. This time Bohr took the matter seriously and asked many questions and in particular, did not attack Jensen. Bohr's no negative attitude encouraged Jensen to listen more carefully to Haxel and Suess and to take their findings seriously. Thus, Jensen and his colleagues were driven to look for an agent that would affect the order of the energy levels so as to fit the nuclear levels and they right away hit the S-L interaction. But the dramatic results were apparently too incredible for some people to digest and the first version of the paper was rejected by the editor *of a serious journal*[24] claiming that: *It is not really physics but rather playing with numbers.* Jensen then sent the paper to Weisskopf, who forwarded it to the Physical Review, which published it two weeks before Mayer's paper was published. It should be recalled that Weisskopf was by then a leading authority in nuclear physics, published many results within the framework of the compound nucleus. However, Weisskopf was fast to recognize the discovery of Jensen and his colleagues. Yet, Jensen was not confident in the physicality of his results until he presented them in front of Bohr in Copenhagen. This time Bohr agreed that *the shell model explains why many nuclei do not show energy levels of rotation.*

12. The final vindication - From three remains one

The revival of the shell model and the agreement with observation started to convince physicists in the viability of the shell model . It was still far from perfection and next approximations started to appear, in particular with respect to finer details, like the quadrupole moment of nuclei. For example, Townes, Foley & Low (1949), who pointed that in order to explain the quadrupole moments a mixture of the single nucleon state must be mixed with some nucleon states from the core. In other words, it is not a compound or a shell model but in between.

With three models on the table, comparisons with the experimental data were very quickly carried out by several groups. Hughes & Couteur (1950) showed that ^{5}He agrees with Mayer's scheme and ordering of the levels and does not agree with the schemes of Feenberg and Hammack and Nordheim. On the other hand, Bethe and Butler (1952) were not yet convinced that the shell model is correct and proposed a critical test which should

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Goldschmidt died in England in 1947 just before the shell model festival started.

[23] The Geochemical Society, which considers Goldschmidt as the founder of modern geochemistry and crystal chemistry, established the Goldschmidt Medal. This paper was the address Suess delivered when he was awarded the Goldschmidt medal.

[24] The journal was Nature.

give information on how accurate is the picture in which nucleons move individually outside the core. Similarly Umezawa and collaborators (Umezawa et al., 1951) found a nice agreement between Mayer's theory and the Fermi theory of β-decay, though they pointed to some problems for $N > 40$. Mayer (1950a) herself added empirical evidence for her model. She calculated the energy levels of a single particle *in a potential between that of a three dimensional harmonic oscillator and a square well* . Next, Mayer (1950b) added more theoretical considerations when she demonstrated the S-L coupling model agrees well with the idea of a very short range nuclear force. Béné (1952) showed that Mayer's scheme provides a nice agreement with observed nuclear magnetic moments.

On May 1953, a conference on Nuclear Spectroscopy and the Shell Model took place in Indiana University and the different schemes were compared. Many technical details were discussed but no final verdict was published. Somehow each side had an answer to all problems. Note that by now the shell model gained an unofficial recognition as bona fide physics. The 1963 Nobel prize in Physics was awarded 1/2 to Wigner *for contributions to nuclear physics, elementary particles and symmetry principles in physics*, *for the discoveries concerning nuclear shell structure*, a 1/4 to Mayer (1906-1972)[25] and a 1/4 to Jensen (1907-1973). One can say that the Nobel prize was awarded to those who expanded on the initial work of Elsasser and dared to question the prevailing dogma. The irony has two folds: The Nobel prize was first awarded for the ′shell model′, which Niels Bohr detested and preached vehemently against, before it was awarded in 1975 to Niels Bohr's son Aage Bohr (1922-2009), Ben Mottelson and Leo Rainwater (1917-1986) for the 'collective model' his father initiated and followed by so many. We discussed at length the shell model because of its dominant role in the synthesis of the elements. However, the collective or the unified model is very successful in describing dynamic properties which result from collective phenomena like rotation of nuclei. In 1953 Bohr and Mottelson (1965) developed the theory of the collective model, in particular the rotation of the nuclei. In 1953 Temmer and Heydenburg (1954) discovered such nuclear levels, a discovery which signaled a victory for the collective model. But the observational evidence of the magic numbers was not washed away by this discovery. So attempts to incorporate single particle level (into a ′unified′ model) began. In this way Nilsson and Mottelson (1955) managed to predict the magic numbers from the united model.

13. How Nature teases physicists and astrophysicists

The nucleus ^{56}Ni is a doubly magic nucleus ($N = 28$ and $Z = 28$) but against all expectations of any nuclear model, is unstable and decays through electron capture to ^{56}Co and next to ^{56}Fe. We mention here this case because the

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[25] You would not believe it: a Nobel prize winner without a permanent position. Maria Mayer, born Goeppert, married Joseph Mayer in 1930 in Germany and moved to the ′New World′, where the chemist Joseph got a position in Johns Hopkins University. Maria however, did not get a paid position because of Anti-Nepotism Rules. She became a volunteer associate. In this position she could do research. From Johns Hopkins the Mayers moved to Columbia, New York where Maria got a part-time teaching job in Sarah Lawrence College, a liberal arts college for women. From New York the Mayers moved to the University of Chicago and Mayer was still without a paid job. It was there that she met Teller and collaborated with him on his polyneutron idea for the formation of the elements, an idea which simply did not work. On the other hand, she was exposed to the real element abundances and started to work alone on the shell model. In 1960, three years before she was recognized by the Nobel committee, she was offered a full professorship at the new campus of the University of California at San Diego. Joseph Mayer was appointed a professor in the Chemistry Department. In California like in California, the university authorities were sufficiently clever and flexible to overcome the anti-nepotism-woman-discriminating rule and offered Maria, and her husband, positions. They repeated the procedure when astrophysicist Jeff and astronomer Margaret Burbidge were attracted to San Diego. The astronomer Margaret got a position in chemistry.

energy released in the decay, 6.702MeV per nucleus of $^{56}Ni_{28}$, heats the exploding supernova and causes it to shine. It is this unexpected anomaly which provides the energy for the supernova type I and allows us to observe it. Consequently, the total light energy emitted by the supernova is proportional to the total mass of ^{56}Ni formed in the explosion. The liquid drop model in this form did not provide satisfactory results for the binding energies of nuclei, Weizsäcker proposed an empirical way to calculate the nuclear binding energies. What he actually did was to take the terms dictated by the liquid drop and add corrections due to single nucleons effects as demonstrated by Elsasser. A general formula was written and in it three unknown numerical constants. The constants were found by fitting the formula to the data. Beware, it is not a genuine prediction or explanation of the stability of nuclei or why there are such nuclei because the formulae on the basis of which the conditions were derived, is a phenomenological one.

Many researchers contributed to improve the original Bethe-Weizsäcker total mass formula and today there are several hundreds of such formulae and the best once provided an error of 1.6-3.5MeV per nucleus. While relative error is quite small, for example for a nucleus with $A = 100$, the relative error is $\sim 2 \times 10^{-5}$. **This is far from being sufficient for stellar neutron capture processes, for an accuracy better than 1/2 MeV is needed.** Despite the exponential increase in computer power the fundamental problem remained uncracked, namely the derivation from first principles of nuclear structure in general and the shell model in particular, is still impossible. You cannot today assume a nuclear force and derive from it the binding energies of the nuclei. The problem is so complicated that we even do not know how sensitive the overall properties of the shell model are to the fine details of the assumed effective interactions. It might very well be that physicists have not yet discovered the right approach to attack the problem of a large number of nucleons, but not sufficiently large number that statistical approaches become valid.

14. The B^{2}FH paper

Burbidge, Burbidge, Fowler & Hoyle (1957) published a seminar paper about the synthesis of the elements. The paper became a road map to nuclear astrophysics. The authors drew the schematic shape of the abundance curve, cf. fig. 2. The arrow mark the peaks. B2FH introduced the notation and baptized the s process (slow neutron capture) and the r process (the fast neutron capture). By fast we mean so fast that the capturing nucleus does not have time to disintegrate. The idea for the r process came from the discovery of ^{252}Cf in the debris of the American H-bomb test. In the r-process, there is no time for β decays and neutron absorption continues until the $\gamma' s$ from the hot radiation fields ($T \sim 2 - 5 \times 10^9$K) do not allow an additional neutron capture and let the nucleus undergo a β-decay. The time-scale 1sec.

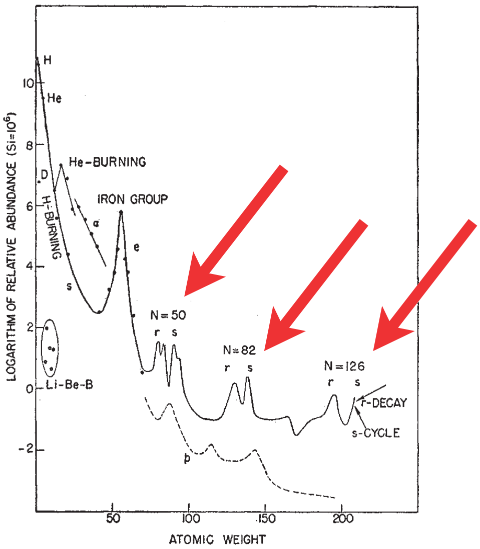

Fig. 2. The schematic cosmic abundance curve as formulated by B^{2}FH.

15. So what are the problems with the heavy elements

We consider here only the r-process. The r-process proceeds along the borders of the stability valley where most of the properties of the elements and isotopes are simply not known, in particular for large A's. Moreover, the nuclear physics changes its nature, new phenomena emerge and extrapolation becomes questionable.

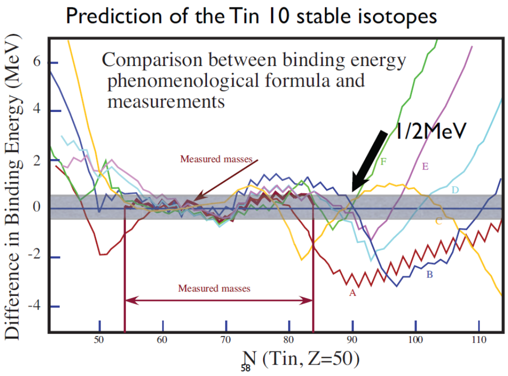

Fig. 3. The nuclear binding energy in the neutron number proton number plane. The red arrows mark the particularly tightly bound nuclei.

In fig. 4 we show the situation vis a vis the nuclear data for the heavy elements. Only a very narrow strip, quite close to the minimum of the valley of stability has been explored to far. The majority, called terra incognita, is simply unknown and in todays calculation is extrapolated. Is this extrapolation justified? We remind that the r-process advances along isotopes with life time of a fraction of a second. The research in the properties of isotopes of heavy elements, has not reached such short-lived nuclei.

15.1. The astrophysical site

No astrophysical site was identified as a producer of all the r-process elements. It is assumed that somehow SN do the job. But no single SN was shown to do it. As a consequence, the current popular idea is that many SNe, a multi-event, are needed to produce the observed abundances. Then a best fit to the observation is made. But in checking the number of parameters played with, you find right away that the number of fitting parameters is very

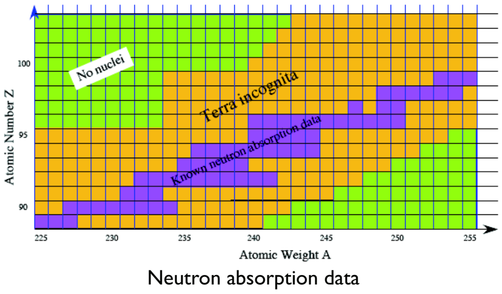

Fig. 4. Nuclei needed for the synthesis but without measured data

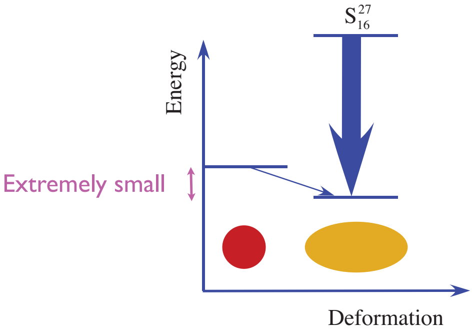

Fig. 5. Nuclei with practically two ground states.

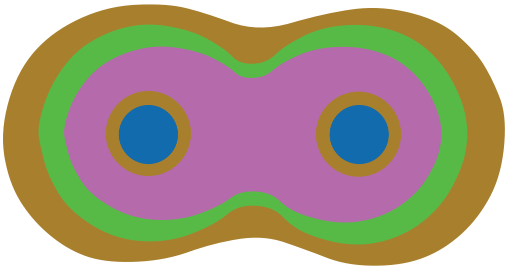

Fig. 6. A typical nucleus with two cores. In this case ^{8}Be.

large (relative to the available data) defeating the idea that the observed abundances is a mix over many SNe. Next, if the *s* and the *r* process are truly independent how come the abundance curve appears smooth. No SN calculation nor any suggested site for the r-process succeeds in predicting the abundances when the nuclear physics problem is combined with the astrophysical one. Recall, we still do not know how

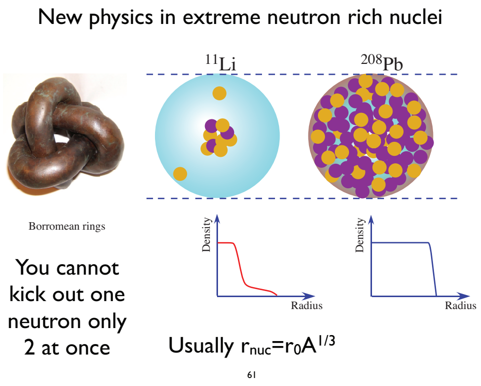

Fig. 7. The structure of neutron rich nuclei: The neutron-halo nuclei are sometimes called Borromean nuclei because you cannot kick out a single neutron. All attempts end with two neutrons ejected.

SNe explode. The amount of energy required to synthesize the heavier than iron nuclei is minute and the total amount turned them into trace elements. Hence in any SN explosion the synthesis of these elements constitutes a side issue.

15.2. The nuclear physics

The r-process takes place far away from the stability line where the physics is poorly known. We mention here several problematic topics.

The Magic Numbers change. Several experiments indicate that the nature of the magic numbers change for neutron rich nuclei and one cannot trust the idea that they are universal.

The mass formula: There are about 2000 different expressions for the mass formulae. All formulae have a large number of parameters which are fitted to known data. We do not know how good the different formulae are in predicting the masses of nuclei with unknown masses but we can compare the predictions by different formulae. Such a comparison is shown in fig. 6 along with the fit to the Tin isotopes. As can be easily verified, all formulae provide an equally good fit to the observed data but they diverge outside the fitting domain. This is a well known phenomenon, fitting formulae can yield very poor results outside the fitting domain because they are not constraint there. The accuracy required by the mass formula is better than half an MeV while the deviations between the formulae is much more than that.

The nature of the physics: The physics changes with increased number of neutrons. In fig. 5 we show a new phenomenon with appears in neutron rich nuclei, namely ′two ground states′ practically at the same energy appear. In certain nuclei the nucleus splits into two separate cores. Last, neutron rich nuclei start to develop for sufficiently high N/Z, a halo of neutron which behaves as neutron matter.

In summary, the properties of neutron rich nuclei far away from the stability line are quite different from the those of the nuclei we are used to work close to the minimum of the stability valley. But far away from the stability, the nuclei expose a plethora of new and interesting phenomena and defeat simple minded extrapolations. A new and wonderful nuclear physics emerges and must be explored before we attempt to apply the old and probably not so relevant, nuclear physics, to the problem of the synthesis of the heavier than iron nuclei.

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